TURBULENCE ENERGY BALANCE IN REACTING TURBULENT FLOWS

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The turbulence accompanying combustion and the propagation of detonation waves in gases has been studied theoretically and experimentally in many papers [1-8]. The attention of researchers has been concentrated on essential questions like how the turbulent flow field interacts with the kinetics of the chemical reaction and to what extent the process of chemical change is intensified, and how the turbulence itself is deformed by the heat released and the accompanying expansion of the gases. The various mechanisms proposed for these phenomena are based on various hypotheses concerning the structure of the combustion zone and the determinative stage of the interaction of the turbulence with the chemical-reaction kinetics. The mechanism of turbulence generation by combustion proposed in a number of papers [3-6] is based on the observation in turbulent flow of a weakly curved flickering laminar flame. This gives rise to a nonuniform flow field of the gas, part of the energy of which goes over into the energy of turbulent fluctuations. Other authors [7, 8] considered the turbulence field to interact with the chemical-reaction kinetics via a volume mechanism and suggested a criterion of turbulence intensification based on certain physical considerations, e.g., the condition for the intensification of thermogaskinetic oscillations proposed by Rayleigh [9]. In the present paper the problem is analyzed by introducing Kolmogorov's general equation for the turbulence energy balance in reacting turbulent flows [10]. In accordance with this equation the turbulence energy can vary due to energy exchange between the turbulent motion and the mean gas flow as a result of the work on turbulent mass transport in the acceleration field of the mean flow, and due to the effect of pressure fluctuations on the rate of thermal expansion from the chemical reaction. Each of these effects is considered and analyzed.

1. We commence from the equation of motion and continuity of a compressible viscous fluid,

$$\frac{\partial(\rho u_i)}{\partial t}+\frac{\partial}{\partial x_\alpha}\left[\rho u_i u_\alpha + p\delta_{i\alpha} - \sigma_{i\alpha}\right] = 0,\quad i,\alpha = 1, 2, 3; \tag{1.1}$$

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x_\alpha}(\rho u_\alpha) = 0, \tag{1.2}$$

where ρ is the density of the fluid; u_i are the components of velocity; p is the pressure; $\sigma_{i\alpha}$ is the viscous stress tensor; t is time; x_α are the coordinates; $\delta_{i\alpha}$ is the Kronecker delta; $\delta_{i\alpha} = 1$ for $i = \alpha$; $\delta_{i\alpha} = 0$ for $i \neq \alpha$. Repeated indices imply summation over all coordinates.

We represent all hydrodynamic quantities y in the form of a sum of the mean (in the sense of the mathematical expectation) component <y> and the fluctuation y'. Utilizing the properties of the operation of averaging and carrying out identical transformations on Eqs. (1.1) and (1.2), we obtain the following equation describing the balance of the fluctuation energy per unit volume of the turbulent flow <E> [10-12]:

$$\frac{\partial\langle E\rangle}{\partial t}+\frac{\partial}{\partial x_\alpha}\left[\langle E\rangle\langle u_\alpha\rangle + \langle E u'_\alpha\rangle + \langle p' u'_\alpha\rangle - \langle u'_i\sigma'_{i\alpha}\rangle\right] =$$

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$$= -\left\langle \sigma'_{i\alpha} \frac{\partial u'_i}{\partial x_\alpha} \right\rangle + \left\langle p' \frac{\partial u'_i}{\partial x_i} \right\rangle - \langle\rho\rangle \langle u'_i u'_\alpha\rangle \frac{\partial \langle u_i\rangle}{\partial x_\alpha} -$$

$$- \langle\rho' u'_i\rangle \langle u_\alpha\rangle \frac{\partial \langle u_i\rangle}{\partial x_\alpha}, \quad \langle E\rangle = \langle \rho u'_i u'_i\rangle / 2. \tag{1.3}$$

The terms on the right side of Eq. (1.3) play the role of volume sources (sinks) of fluctuation energy; the expression in square brackets on the left side of (1.3) describes the redistribution of the energy of the turbulent motion in space. The first term on the right side is connected with the dissipation of the energy of the turbulent motion due to molecular viscosity. The second term, which characterizes the work by pressure fluctuations in the turbulent expansion (compression) of the gas, is nonzero only in the case of a compressible gas. If there is no chemical reaction giving rise to the thermal expansion of the gas, then positive pressure fluctuations ($p' > 0$) lead to the compression of the gas $\left(\partial u'_\alpha / dx_\alpha < 0\right)$, so that the overall sign of this term is negative. If, however, an exothermal reaction takes place in the gas, positive pressure fluctuations can increase and negative decrease the rate of the reaction and, correspondingly, change the expansion of the gas so that the sign of the term p' $\left\langle p' \frac{\partial u'_\alpha}{\partial x_\alpha} \right\rangle$ may be positive, which corresponds to the action of a source of fluctuation energy.

The third term on the right side, which characterizes energy exchange between the turbulent motion and the mean motion of the gas, is essentially connected with the kinematic picture of the mean motion. For example, in the case of a one-dimensional mean motion, an exothermal reaction leads to the expansion and the acceleration of the flow and part of the energy of the turbulent motion goes over into kinetic energy of the mean motion ($-\langle\rho\rangle \langle u'^2\rangle d \cdot \langle u\rangle / dx < 0$; for definitness it is assumed here and below that the gas flow moves in the direction of positive x).

Finally, the last term on the right side is connected with the work on turbulent mass transport in the acceleration field of the mean flow. Since in turbulent flows with combustion the density fluctuations are determined primarily by the temperature fluctuations, it follows that a positive velocity fluctuation, which brings colder gas to the point under consideration, leads to a positive density fluctuation. Accordingly, in the simplest case of a one-dimensional mean flow, this term also plays the role of a sink of turbulent energy ($-\langle\rho' u'\rangle \langle u\rangle d\langle u\rangle / dx < 0$).

2. As an example of the interaction of the chemical reaction kinetics with the turbulent fluctuations of a gas flow we consider combustion in a homogeneous chemical reactor and the so-called induction mode of combustion. We consider a developed turbulent gas flow in which transport of momentum, matter, and energy by the fluctuations occurs much more intensively than molecular transport porcesses, so that the terms containing $\sigma'_{i\alpha}$ can be neglected in Eq. (1.3).

A homogeneous chemical reactor in which an exothermal reaction takes place is the simplest example of the interaction of turbulence with a reacting medium. In this case all the properties of the turbulent motion are identical over the whole volume of the reactor, which permits of the maximum possible simplification of the equation describing the fluctuation energy. Integrating Eq. (1.3) over the volume of the reactor gives

$$m(\langle e\rangle - \langle e\rangle_0) = \left\langle p' \frac{\partial u'_i}{\partial x_i} \right\rangle V. \tag{2.1}$$

Here m is the mass flow rate through the reactor; V is the volume of the reactor; and $\langle e\rangle = \langle u'_i u'_i\rangle / 2$ is the energy of the turbulent motion of unit mass of gas. The index 0 refers to the value of the fluctuation energy at the reactor inlet.

In order to calculate the work of the pressure fluctuations we return to the balance equation for the thermal energy of a compressible gas and write it, utilizing the equation of state of a gas $p = \rho RT$ (R is the gas constant), in the form

$$\frac{k}{k-1} p \frac{\partial u_i}{\partial x_i} = q\Phi(T, a) - \frac{1}{k-1}\left(\frac{\partial p}{\partial t} + u_i \frac{\partial p}{\partial x_i}\right), \tag{2.2}$$

where $k = c_p/c_v$; c_p and c_v are the specific heats of the gas at constant pressure and constant volume; q is the thermal effect of the reaction; $\Phi(T, a)$ is the rate of the reaction; T is the gas temperature; and a is the concentration of the reacting component in the gas. The molecular thermal conductivity is omitted in Eq. (2.2) as we are considering the case of developed turbulence. A relationship determining the work of the pressure fluctuations can be obtained from (2.2). To this end we perform two averaging operations: first of all, we average Eq. (2.2), and then we average the equation obtained by multiplying (2.2) by p'. Solving the obtained relationships for the sought-for quantity and neglecting the second power of the relative pressure fluctuation $p'/\langle p\rangle$, which is proportional to the Mach number of the turbulent motion, compared with the first, we obtain

$$\left\langle p'\frac{\partial u'_i}{\partial x_i}\right\rangle = \frac{k-1}{k}\frac{q}{\langle p\rangle}\langle p'\Phi'\rangle. \tag{2.3}$$

The fluctuations of the heat-release function Φ' are connected with the fluctuations in the temperature and in the concentration of the reacting substance. At large activation energies of the reaction the effect of concentration fluctuations is negligible compared with the effect of temperature fluctuations. We restrict the discussion to sufficiently small temperature fluctuations and utilize the linear representation $\Phi' = (d\Phi/dT)T'$; to determine $T'(t)$ we consider the linearized equation of thermal conductivity in Lagrangian coordinates:

$$c_p\langle\rho\rangle\frac{dT'}{dt} - \frac{dp'}{dt} = q\frac{d\Phi}{dT}T'. \tag{2.4}$$

To calculate the correlation $\langle p'T'\rangle$ we make use of the ergodic hypothesis: a time average at a fixed point in space we replace by a time average at each fixed particle with a subsequent average over the total time the various particles remain in the reactor.

We represent the pressure fluctuations at a particle of material in the form $p' = \sum\limits_{n=1}^{\infty} p_n \sin n\omega t$, where p_n are the coefficients of expansion of p' in a trigonometric series and ω is the frequency of the pressure fluctuations. With this expression of p' we solve (2.4) for the temperature fluctuations and obtain, subject to the initial condition $T' = 0$ at $t = 0$,

$$\frac{T'}{\langle T\rangle} = \frac{k-1}{k}\sum_{m=1}^{\infty}\frac{p_m}{\langle p\rangle}\frac{ms}{1+m^2s^2}\left\{\exp\left(\frac{t}{t_c}\right) - \cos m\omega t + ms\sin m\omega t\right\}.$$

Here $t_c = c_p\langle\rho\rangle q(d\Phi/dT)$ is the characteristic time of heating of the gas from the chemical reaction, and $s = \omega t_c$.

For an individual particle spending a time t_0 in the reactor we have

$$\begin{aligned}\int_0^{t_0} p'(t)\,T'(t)\,dt = {}&\frac{k-1}{k}\langle p\rangle\langle T\rangle\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{p_n p_m}{\langle p\rangle^2}\frac{mst_c}{1+m^2s^2}\times\\ &\times\left\{\frac{1}{1+n^2s^2}\sin n\omega t_0\exp\left(\frac{t_0}{t_c}\right) - \frac{ns}{1+n^2s^2}\left[\exp\left(\frac{t_0}{t_c}\right)\times\right.\right.\\ &\left.\times\cos n\omega t_0 - 1\right] + \frac{1}{2s}\left[\frac{1}{n+m}(\cos(n+m)\omega t_0 - 1) - \right.\\ &\left.\left.- \frac{1}{n-m}(\cos(n-m)\omega t_0 - 1)\right] + \frac{m}{2}\left[\frac{1}{n-m}\sin(n-m)\omega t_0 - \frac{1}{n+m}\sin(n+m)\omega t_0\right]\right\}.\end{aligned} \tag{2.5}$$

Due to the turbulent character of the motion of the gas different particles spend different times in the reactor. In order to average relationship (2.5) over t_0 and obtain the correlation function $\langle p'T'\rangle$ we require to know $F(t_0)$, the probability density distribution over times t_0 spent in the reactor. Let us suppose that $F(t_0)$ conforms to the normal law, i.e.,

$$F(t_0) = (\pi\sigma)^{-1/2} \exp\left[-\frac{(t_0 - \langle t_0\rangle)^2}{\sigma}\right], \tag{2.6}$$

where $\langle t_0 \rangle$ is the mean time spent by the gas in the reaction; σ is the dispersion of the distribution law; and the factor $(\pi\sigma)^{-1/2}$ is chosen from the normalization condition

$$\int_{-\infty}^{+\infty} F(t_0)\, dt_0 = 1. \tag{2.7}$$

The limits of integration in (2.7), as in the calculation of subsequent integrals containing $F(t_0)$, are taken to be infinite. This approximation represents the asymptotic limit for $\langle t_0\rangle^2/\sigma \gg 1$, when the characteristic spread of the time spent in the reactor is much less than the mean time $\langle t_0 \rangle$, so that the natural time segment of the problem $[0, \langle t_0 \rangle]$ for integration is "infinitely large."

Averaging relationship (2.5) with respect to t_0 for density distribution (2.6) leads to the appearance of integrals of the form

$$(\pi\sigma)^{-1/2} \int_{-\infty}^{+\infty} \sin[(n+m)\omega t_0] \exp\left[-\frac{(t_0 - \langle t_0\rangle)^2}{\sigma}\right] dt_0 = \exp\left[-\frac{(n+m)^2\omega^2\sigma}{4}\right] \sin[(n+m)\omega\langle t_0\rangle],$$

the values of which are strongly dependent on the parameter $\omega^2\sigma$. By virtue of the stochastic character of developed turbulence we shall consider the characteristic spread of the time spent in the reactor $(\sigma)^{1/2}$ to span sufficiently many turbulent fluctuations $\omega^2\sigma \gg 1$. On going, after averaging, to the limit $\omega^2\sigma \to \infty$, we obtain

$$\frac{\langle p'T'\rangle}{\langle p\rangle\langle T\rangle} = \frac{1}{\langle t_0\rangle} \int_{-\infty}^{+\infty} dt_0 \left(\int_0^{t_0} p'(t)\, T'(t)\, dt \right) = \frac{k-1}{k} \sum_{\substack{n=1 \\ n\neq m}}^{\infty} \sum_{m=1}^{\infty} \frac{p_n p_m}{\langle p\rangle^2} \times$$

$$\times \frac{ms^2}{1+m^2s^2} h \left[\frac{n}{1+s^2} - \frac{1}{2s^2(n+m)} + \frac{1}{2s^2(n-m)}\right] + \frac{k-1}{k} \sum_{n=1}^{\infty} \frac{p_n^2}{\langle p\rangle^2} hU_n, \tag{2.8}$$

where

$$h = t_c/\langle t_0\rangle, \quad U_n = \frac{ns^2}{1+n^2s^2}\left[\frac{n}{1+s^2} - \frac{1}{4ns^2} + \frac{n}{2}h^{-1}\right].$$

The result of the averaging, (2.8), does not depend on the dispersion of the normal distribution law. It can be shown that in the limit $\langle t_0\rangle^2/\sigma \gg 1, \omega^2\sigma \gg 1$ the result of the averaging is also independent of the form of the probability density distribution.

At high frequencies of the turbulent fluctuations, when the time of heating of the gas t_c is much greater than the period of the turbulent fluctuations $\omega^{-1}(s \gg 1)$, we obtain from (2.8) a relationship corresponding to an adiabatic dependence of the gas temperature on the pressure fluctuations (the chemical reaction does not have time to exert any significant effect):

$$\frac{\langle p'T'\rangle}{\langle p\rangle\langle T\rangle} = \frac{k-1}{2k} \sum_{n=1}^{\infty} \frac{p_n^2}{\langle p\rangle^2}.$$

A more simple expression is obtained from (2.8) if the pressure fluctuations are assumed to have only a single frequency (a degenerate spectrum). In this case only a single term with n = m = 1 remains in the series in (2.8):

$$\frac{\langle p'T'\rangle}{\langle p\rangle\langle T\rangle} = \frac{k-1}{k}\left(\frac{p_1}{\langle p\rangle}\right)^2 h\,\frac{s^2}{1+s^2}\left[\frac{1}{2}h^{-1} + \frac{1}{1+s^2} - \frac{1}{4s^2}\right]. \tag{2.9}$$

In the general case, in order to calculate the correlation <p'T'> we require to know the frequency spectrum of the pressure fluctuations in the turbulent flow. Use may also be made of the spectrum of the velocity fluctuations of the flow, bearing in mind the following relationship for isotropic and homogeneous turbulence between the amplitudes of the pressure and the velocity fluctuations:

$$p' = g\langle\rho\rangle u'^2, \tag{2.10}$$

where the coefficient of proportionality $g \approx 0.7$. This relationship derives from theoretical and experimental investigations [13].

We turn now to the relationships describing mass and thermal-energy balance in the reactor:

$$m = \langle\rho\rangle V/\langle t_0\rangle; \tag{2.11}$$

$$mc_p(\langle T\rangle - T_0) = Vq\Phi. \tag{2.12}$$

In Eq. (2.12) the thermal energy converted into the energy of turbulent motion is ignored; estimates show that these losses are small compared with the total store of heat in the reactor.

Utilizing mass balance equation (2.11), the turbulence-energy balance equation (2.1), and relationships (2.3), (2.9), and (2.10), we obtain the following expression for the energy of the turbulent motion in a reactor in the case of pressure fluctuations with a single frequency:

$$\frac{\langle e\rangle}{\langle e\rangle_0} = \frac{2}{\Gamma}\left[1 - (1-\Gamma)^{1/2}\right]), \tag{2.13}$$

where

$$\Gamma = \frac{8}{3}g^2(k-1)M^2U_1(s,h);$$

$$U_1 = \frac{s^2}{1+s^2}\left[\frac{1}{1+s^2} - \frac{1}{4s^2} + \frac{1}{2}h^{-1}\right];$$

$$\mathrm{M} = (\langle e\rangle_0)^{1/2}/(k\langle p\rangle/\langle\rho\rangle)^{1/2};$$

and M is the Mach number of the turbulent motion of the initial flow.

The variation of the energy of the turbulent motion in the reactor depends on the frequency of the pressure fluctuations through the parameter $s = \omega t_c$. If $h > 1.5$, the function U_1 determining the dependence on frequency has a maximum at $s = s_* = [(5h+3)/(3h-2)]^{1/2}$. If $h < 1.5$, the function U_1 increases monotonically with increasing s, reaching at large s the limiting value $U_1 = {}^1/{}_2h^{-1}$. The dependence on frequency of the intensity of generation of turbulence energy leads to a deformation of the frequency spectrum of the fluctuation velocity of the initial turbulent flow.

The heat balance equation imposes a constraint on the region of variation of the parameter h. Differentiating (2.12) with respect to <T>, we obtain h = 1.

By (2.13) the maximum twofold increase of the energy of the turbulent motion is achieved at $\Gamma = 1$. For $\Gamma > 1$ no steady modes of turbulent motion inthe reactor exist. The disruption of the steady mode is connected with the nonlinear dependence of the work of the pressure fluctuations on the turbulenece energy in the reactor $\left(\left\langle p'\frac{\partial u'_i}{\partial x_i}\right\rangle \sim \langle e\rangle^2\right)$, whereas the removal of the energy of the turbulent motion from the reactor depends on it linearly.

For $g \approx 0.7$, $k = 1.4$ and $s \gg 1$ the value of the parameter $\Gamma = 0.26\,\mathrm{M}^2$ and, by (2.13), the increase of the energy of turbulent motion becomes significant only at velocities of turbulent motion close to sonic. Thus, for example, for M = 1 we obtain $\Gamma = 0.26$, $\langle e\rangle/\langle e\rangle_0 = 1.15$. Under conditions of turbulent flame propagation the turbulent velocities are usually much less than sonic, with the result that the generation of turbulence in flames due to the work of the pressure fluctuations can be neglected.

3. In order to illustrate the effect of spatial inhomogeneity of the mean flow on the balance of the fluctuation energy in reacting flows we consider a steady on-average-one-dimensional turbulent flow in which the gas is accelerated due to an influx of heat from a volume chemical reaction. Simplest of all is the so-called "induction mode of combustion" [14], in which the initial velocity of the flow is so large that convective transport of heat and matter greatly exceeds transport due to turbulent thermal conductivity and diffusion, so that the latter can be neglected.

In order to investigate the effect of spatial inhomogeneity it is necessary to know the spatial distribution of the mean flow velocity, the mean temperature, and the mean gas density. In calculating the mean values we utilize the fact that the fluctuations of the hydrodynamic quantities in a turbulent flow amount to a small fraction of the mean values, so that the equation of the fluctuation energy has a higher order of smallness than the equations for the mean values of velocity, temperature, and density. Accordingly, when calculating from the latter equations the coefficients before the fluctuating components in the fluctuation-energy balance equation, we can neglect the influence of turbulent effects on the distribution of the mean quantitities.

In the induction mode the concentration and temperature fields are similar, with the result that the heat-release function proves to be dependent on temperature alone. Integrating the equation of thermal conductivity subject to the condition $\langle T\rangle = \langle T\rangle_0$ at $x = 0$ we obtain

$$x = [\langle\rho\rangle_0 \langle u\rangle_0 c_p/q] \int_{\langle T\rangle_0}^{\langle T\rangle} [\Phi(\langle T\rangle)]^{-1} d\langle T\rangle. \tag{3.1}$$

Knowing the temperature distribution, we can use the equation of state

$$\langle\rho\rangle / \langle T\rangle = \langle p\rangle / R = \text{const} \tag{3.2}$$

to determine the density distribution, and the equation of continuity

$$\langle\rho\rangle\langle u\rangle = \langle\rho\rangle_0\langle u\rangle_0 = \text{const} \tag{3.3}$$

to determine the mean velocity distribution.

The index 0 refers to the values of the parameters of the initial flow.

Formula (3.1) determines a characteristic spatial scale x_* if we substitute into it a characteristic temperature (e.g., the temperature of combustion).

Let us now analyze Eq. (1.3) describing the balance of the fluctuation energy. We close it by invoking hypotheses that can be regarded as the linear expansion of the correlations as a series in the gradients of the mean quantities [;3]

$$\langle e u_1'\rangle = -\nu_1 L(\langle e\rangle)^{1/2} d\langle e\rangle/dx; \tag{3.4}$$

$$\langle\rho' u_1'\rangle = -\nu_2 L(\langle e\rangle)^{1/2} d\langle\rho\rangle/dx; \tag{3.5}$$

$$\langle\rho\rangle\langle u_1' u_1'\rangle = \frac{2}{3}\langle\rho\rangle\langle e\rangle. \tag{3.6}$$

Relationship (3.6) is a particular case of the more general relationship

$$\langle\rho\rangle\langle u_i' u_j'\rangle = \frac{2}{3}\langle\rho\rangle(e)\delta_{ij} - \nu_3 L(\langle e\rangle)^{1/2}\left[\frac{\partial\langle\rho u_i\rangle}{\partial x_j} + \frac{\partial\langle\rho u_j\rangle}{\partial x_i}\right],$$

$$\delta_{ij} = 1,\ i = j;\ \delta_{ij} = 0,\ i \neq j,$$

characterizing the connection between the turbulent stress tensor and the strain tensor in a turbulent flow [13].

It is assumed in these relationships, as in [10-12], that the coefficients of turbulent transport of fluctuation energy in (3.4) and mass in (3.5) are proportional to the square root of the fluctuation energy and to the scale of the turbulence L; ν_1, ν_2, ν_3 are dimensionless proportionality coefficients.

We shall assume that the local properties of homogeneity and isotropy of the turbulent field are conserved during combustion. Accordingly, the correlation between the pressure and the velocity fluctuations $\langle p'u_i' \rangle$ may be neglected.

We bring the fluctuation-energy balance equation (1.3) and the closure hypotheses (3.4)-(3.6) to dimensionless form by choosing as scale values for $\langle e \rangle$, $\langle u \rangle$, $\langle \rho \rangle$, and x their characteristic values $\langle e \rangle_0$, $\langle u \rangle_0$, $\langle \rho \rangle_0$, and x_*. We then obtain for a one-dimensional steady flow (the averaging sign over the dimensionless quantities is omitted)

$$\frac{d}{dz}\left(\rho u e - \frac{1}{\mathrm{Pe}} \nu_1 (e)^{1/2} \frac{de}{dz}\right) = -\frac{2}{3} \rho e \frac{du}{dz} + \frac{1}{\varepsilon^2 \mathrm{Pe}} \nu_2 (e)^{1/2} \cdot u \frac{du}{dz} \frac{d\rho}{dz}, \tag{3.7}$$

where $\mathrm{Pe} = \langle u \rangle_0 x_* / L(\langle e \rangle_0)^{1/2}$ is the turbulence Péclet number; $z = x/x_*$, $\varepsilon = (\langle e \rangle_0)^{1/2}/\langle u \rangle_0$ is the intensity of the initial turbulence.

The induction mode of combustion in a turbulent flow is characterized by a large value of the Péclet number. If Pe is so much greater than unity that also $\varepsilon^2 \mathrm{Pe} \gg 1$ (for example, because $x_*/L \gg 1$), then in (3.7) we may neglect turbulent diffusion and the expenditure of fluctuation energy on turbulent mass transport [the second and fourth terms of Eq. (3.7)]. Integrating the correspondingly simplified Eq. (3.7) and remembering the continuity equation (3.3), we obtain the fluctuation energy as a function of the flow velocity: $e = u^{-2/3}$, from which we see that with increasing velocity the fluctuation energy decreases — it goes over into energy of the mean motion. The flow can be accelerated under such conditions in various ways. In the present case of the induction mode of combustion the flow is accelerated due to the heat release in the chemical reaction. In order to calculate the spatial distribution of the fluctuation energy, recourse must be made to Eqs. (3.1)-(3.3).

At small initial turbulence intensities the last term in Eq. (3.7), describing the expenditure of fluctuation energy on turbulent mass transport, begins to play a greater relative role. Considering the aymptotic case of $\mathrm{Pe} \gg 1$, but $\varepsilon^2 \mathrm{Pe} \sim 1$, we can write the solution of Eq. (3.7) subject to (3.3) in the form

$$e(z) = [u(z)]^{-2/3} \left\{1 - \frac{\nu_3}{2\varepsilon^2 \mathrm{Pe}} \int_{-\infty}^{z} [u(x)]^{-2/3} \left(\frac{du}{dx}\right)^2 dx\right\}^2. \tag{3.8}$$

It can be seen from (3.8) that turbulent mass transport in the acceleration field of the mean flow leads to a further reduction of the energy of the turbulent motion.

An approximate computation of the integrals in (3.8) and (3.1) leads to the following relationships describing the dependence of the energy of the turbulent motion on the temperature in the flow and the dependence of temperature on the coordinate:

$$e(\theta) = [\alpha + (\alpha - 1)(\theta/\theta_0)]^{-2/3} \left\{1 - \frac{\nu_2}{4} \frac{1}{(\varepsilon \mathrm{Pe})^2} I(\theta)\right\}^2; \tag{3.9}$$

$$z = \alpha \mathrm{Pe} \{ [\mathrm{Ei}(\theta_0) - \mathrm{Ei}(|\theta|)] - (\exp \theta_0 - \exp |\theta|) \}, \tag{3.10}$$

where $\mathrm{Ei}(x) = \int_{-\infty}^{x} \frac{\exp t}{t} dt$ is the exponential integral;

$$I(\theta) = \left(\frac{\alpha - 1}{\theta_0}\right)^2 \{1 + [\alpha + (\alpha - 1)(\theta/\theta_0)]^{-5/3}\} \{(|\theta| + 1) \exp \theta - (\theta_0 + 1) \exp(-\theta_0)\}; \tag{3.11}$$

$\alpha = \langle T \rangle_k / \langle T \rangle_0$ is the ratio of the temperature of combustion in the flow to the initial temperature; and

$$\theta = E(\langle T \rangle - \langle T \rangle_k)/R \langle T \rangle_k^2; \quad \theta_0 = E(\langle T \rangle_k - \langle T \rangle_0)/R \langle T \rangle_k^2.$$

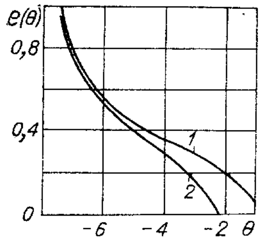

Fig. 1

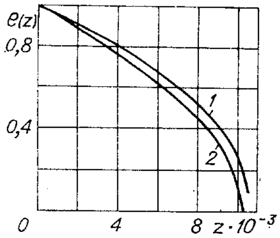

Fig. 2

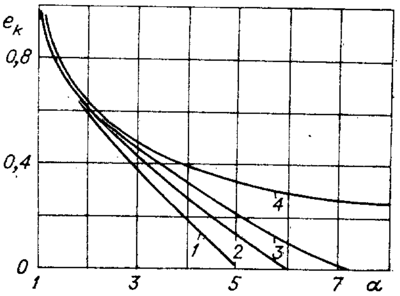

Fig. 3

The derivation of formulas (3.9)-(3.11) utilizes the heat-release function written allowing for the similitude of the concentration and temperature fields and for the variation of the density of the reacting gas:

$$\Phi(\theta) = |\theta|\,(\langle\rho\rangle/\langle\rho\rangle_0)\exp\theta.$$

The characteristic scale x* was taken as $x_* = [L(\langle e\rangle_0)^{1/2} t_*]$; $t_* = \left[\varphi \exp\left(-\frac{E}{R\langle T\rangle_k}\right)\right]^{-1}$ is the time of the chemical reaction at temperature $\langle T\rangle_k$ and at the initial density and concentration of the reacting material; φ is a pre-exponential factor.

On complete combustion of the material the energy of the turbulent motion decreases to a value e_k, given by relationships (3.9) and (3.11) on inserting into them θ = 0:

$$e_k = \left(\frac{1}{\alpha}\right)^{2/3}\left\{1 - \frac{\nu_2}{4}\left(\left(\frac{\alpha}{2}\right)^{1/2}\frac{\alpha - 1}{(\langle e\rangle_0)^{1/2}/\langle u\rangle_*}\right)^2\right\}^2,$$

where $\langle u\rangle_* = \left[L(\langle e\rangle_0)^{1/2}/t_*\right]^{1/2}(2/\alpha)^{1/2}/\theta_0$ is the velocity of normal propagation of a turbulent flame along the reacting gas.

Figures 1 and 2 show plots of the turbulence energy distribution versus temperature and the coordinate in the flow; Fig. 3 shows plots of the final energy of the turbulent motion versus the parameter α for various values of $(\langle e\rangle_0)^{1/2}/\langle u\rangle^*$. Curves 1 and 2 in Figs. 1 and 2 were plotted for $\alpha = 7.7$, $\theta_0 = 7.6$, $\mathrm{Pe} = 10$, $\nu_2 = 1$ for various turbulence intensities of the initial flow $\varepsilon_1 = 0.05$, $\varepsilon_2 = 0.025$. In Fig. 3 curves 1-4 correspond to various values of the parameter $r = (\langle e\rangle_0)^{1/2}/\langle u\rangle_*$: $r_1 = 3{,}6$, $r_2 = 5$, $r_3 = 7$, $r_4 \gg 1$.

Figure 3 shows that if the fluctuation velocity of the initial flow is less than a certain value dependent on α and $\langle u\rangle_*$, then the turbulent flow can go over into laminar right up to complete combustion of the reacting gas.

Transition from a turbulent mode of gas flow to a laminar have been observed in experimental investigations [15]. Such transitions were observed in cases when the flow experienced a sufficiently strong acceleration. The characteristic values of the accelerations were taken as criteria indicating decay of the turbulence. The theoretical analysis made in [15] indicates that laminarization of a turbulent flow is also possible in the case of induction combustion of the fuel gas mixture.

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LITERATURE CITED

1. A. Westenberg, "Measurement of flame turbulence by helium diffusion technique," Vopr. Raketn. Tekh., 26, No. 2 (1955).
2. A. G. Prudnikov, "Measurement by optical-diffusion technique of turbulence of air flows and flames," in: Combustion in a Turbulent Flow [in Russian], Moscow, Izd. Akad. Nauk SSSR (1959), p. 88.

3. A. C. Scurlock and J. H. Grover, "Propagation of turbulent flames," in: Fourth Symposium on Combustion, Baltimore (1953).
4. B. Karlovits, D. W. Denniston, Knapschaefer, and F. E. Wells, "Studies on turbulent flames. A. Flame propagation across velocity gradients. B. Turbulence measurements in Flames," in: Fourth Symposium on Combustion, Baltimore (1953).
5. L. S. Kozachenko, "Effect of flow fluctuations on turbulent velocity of propagation of flames," Énerget. Avtomat., 21, No. 2 (1959).
6. Yukio Misutani, "Amplification of turbulence level by a flame and turbulent flame velocity," Combust. Flame, 19, No. 2, 203 (1972).
7. V. A. Frost, "Variation of turbulence behind combustion zone," Tr. Mosk. Fiz.-Tekh. Inst., 105 (1961).
8. A. Q. Eschenroeder, "Intensification of turbulence by chemical heat release," Phys. Fluids, 7, 1735-1743 (1964).
9. J. W. Rayleigh, The Theory of Sound, Dover Publ. Inc., New York (1945).
10. A. N. Kolmogorov, "The equations of the turbulent motion of an incompressible fluid," Izv. Akad. Nauk SSSR, Ser. Fiz., 6, Nos. 1-2 (1942).
11. A. S. Monin and A. M. Yaglom, Statistical Hydromechanics [in Russian], Izd. Nauka, Moscow (1967).
12. G. I. Barenblatt, "Motion of suspended particles in a turbulent flow," Prikl. Mat. Mekh., 17, No. 3, 261 (1953).
13. J. Hinze, Turbulence, McGraw-Hill (1959).
14. R. M. Zaidel' and Ya. B. Zel'dovich, "Possible modes of steady combustion," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1962).
15. M. M. Nazarchuk, M. M. Kovetskaya, and V. N. Panchenko, Transition of Turbulent Flow Back to Laminar [in Russian], Izd. Naukova Dumka, Kiev (1974).